

Title: Time Evolution in Time-Dependent Systems

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I have been absorbing ideas on new problems to tackle here this past weekend while travelling to Watkins Glen State Park in upstate New York. Having safely arrived home, I thought I would make a mini-summary of the ideas I plan to pursue in the next few posts. Only one of the ideas connects directly to the quantum calculations I've been doing calculations on lately, thus offering a refreshing change in pace to the recent discussion of eigenbasis, linear operators, and probability amplitudes. The ideas I am planning on tackling, in no particular order, are:

- *Differentiate in Time the TDSE:* After having solved for the state vector of the time-dependent 1D QHO problem where $\omega = \omega(t)$ (assuming an initial state in the ground state), it is quite the curiosity to know how the system would respond to a variety of frequency change profiles that range from slow, adiabatic change to rapid, step-like changes in oscillator frequency. Differentiating this TDSE would allow for one to calculate the value of state vector inner products with consideration to the derivative of frequency with respect to time, thus explicitly supplying the information of the scale of change in the potential. This idea may then allow us to understand the change in the state vector when interacting with a time-dependent potential.
- *Large Prime Numbers - a Generalization of the Mersenne Primes:* The largest prime number known today is a Mersenne prime (of the form $2^p - 1$, where p is a large prime number) that has the value $2^{136279841} - 1$ with over 41 million digits. Euclid proved long ago that there are infinity many prime natural numbers, meaning that this Mersenne prime, though the largest *known*, still is dwarfed by the vastness of the positive integer number line. This leaves substantial possibility to find, with the help of computers, a larger prime. In particular, I'd like to examine a set of generalized Mersenne Numbers that have the form $v(n, m, p) = n^p - m^p$ (where n is even / odd, m is odd / even, and p is prime), where the standard Mersenne primes are given by $n = 2$ and $m = n - 1$. Mathematicians have developed a number of very efficient, error free algorithms for judging the primality of these Mersenne prime candidates, known as the generalized Lucas-Lehmer algorithm. Standard Mersenne primes are extraordinarily rare (i.e. very few p produce $v(2, 1, p)$ that are prime), but I am hoping that by not choosing n, m , we may be able to explore a less well-known space of potential prime numbers.
- *The Rationality of the Euler-Mascheroni Number:* I've recently learned about the open problem regarding the idea of rationality or irrationality of the Euler-Mascheroni Number, which is given as $\gamma = 0.577215 \dots$ to 6 significant digits. I have an idea to tackle and examine this problem, starting with one of most well-known definitions:

$$\gamma = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k} - \ln(n)$$

One can imagine expanding the logarithm in powers of $(1 - \frac{1}{n})$, and then applying the binomial Taylor series expansion to each term of the logarithmic expansion. This is an idea that greatly interests me from a series formalism point of view, and strikes deep into the question of what an “infinite” logarithmic series means in this case. Is it less than or greater than the infinite nature of the limit in n ? We will examine this question in the coming weeks.

- *An Interesting Logarithmic Integral Found Online*: Recently, my social media suggested the following interesting integration problem that I’d like to knock off in the next while:

$$I_a = \int_0^\pi dx \ln(1 - 2a \cos(x) + a^2), \text{ where } a > 1$$

This problem caught my eye because of its similarity to patterns I have observed before. For instance:

$$1 - 2a \cos(x) + a^2 = (1 - a \cos(x) + ia \sin(x))(1 - a \cos(x) - ia \sin(x)) = (ae^{-ix} - 1)(ae^{ix} - 1)$$

which is now a product that is easily separable logarithmically. Now, I have to still work out what the next step is to compute the integral of the logarithm of the complex exponentials. To do for the future!

These are my future directions at the moment. I wish everyone a great week! This concludes this blog post.